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# MULTIMEDIA UNIVERSITY

## FINAL EXAMINATION

TRIMESTER 1, 2018/2019

### EEL1176 – FIELD THEORY

( All sections / Groups )

22 OCTOBER 2018

9.00 a.m - 11.00 a.m

( 2 Hours )

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#### INSTRUCTIONS TO STUDENTS

1. This question paper consists of 8 pages with 4 questions only.
2. Answer all **FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the Answer Booklet provided.

**Question 1 (25 marks)**

(a) Given vector  $\vec{A} = y\hat{x} + (x+z)\hat{y}$  and point P is located at  $(-2, 6, 3)$ . Determine the following:

(i) Express the location of point P in spherical coordinates.

[3 marks]

(ii) Evaluate vector  $\vec{A}$  at point P in the spherical system.

[7 marks]

(b) A structure with the following dimensions are provided:  $2 \leq r \leq 6$ ;  $\frac{\pi}{4} \leq \varphi \leq \pi$ ;  $z = 0$ . Determine the total surface area by performing surface integral.

[4 marks]

(c) Given a vector field  $\vec{D} = 3R^2 \hat{R}$ . The flux  $\oint_S \vec{D} \cdot d\vec{s} = 180\pi$  for the region enclosed between the spherical shells defined by  $R = 1$  and  $R = 2$  as seen in Figure 1 (c). Answer the following:

(i) State the Divergence Theorem mathematically.

[1 mark]

(ii) Verify the Divergence theorem by determining the volume integral of the divergence  $\iiint_V \nabla \cdot \vec{D} dv$  for the given region.

[10 marks]

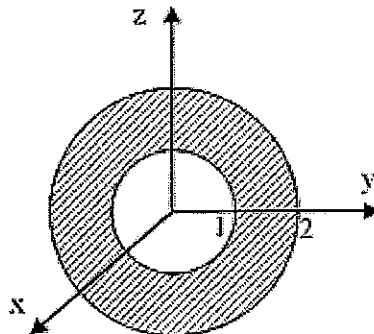


Figure 1(c)

Continued ...

**Question 2 (25 marks)**

- (a) A positively charged rod is brought close to a neutral piece of paper. The charged rod attracts the paper. Draw a diagram showing the separation of charge in the paper, and explain why attraction occurs. [4 marks]
- (b) Two point charges  $Q_1 = 3 \mu\text{C}$  and  $Q_2 = -4 \mu\text{C}$  are placed at  $(3, 2, 1)$  and  $(-4, 0, 6)$ , respectively.
- (i) Determine the force,  $\vec{F}$  on the charge  $20 \mu\text{C}$  at the origin  $(0, 0, 0)$ . [4 marks]
- (ii) Find the electric field intensity,  $\vec{E}$  on the charge  $20 \mu\text{C}$  at the origin  $(0, 0, 0)$ . [2 marks]
- (c) A spherical shell of radius  $a$  shown in Figure 2(c) contains a uniform charge density of  $\rho_v / 2 \text{ (C/m}^3\text{)}$ . Determine the electric flux density,  $\mathbf{D}$  by the given Gaussian surfaces for each of the following cases:
- (i)  $0 < R_1 < a$  [5 marks]
- (ii)  $R_2 > a$  [5 marks]

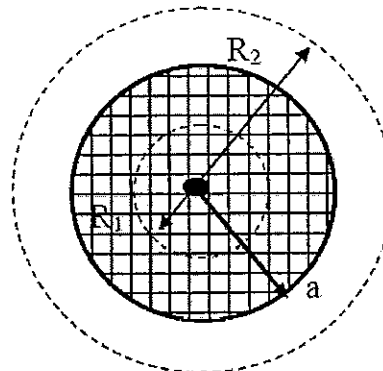


Figure 2(c)

- (d) Figure 2(d) shows 2 parallel plate capacitors made of paper and plastic. Given the paper dielectric constant  $\epsilon_{r1} = 3.7$  and the plastic dielectric constant  $\epsilon_{r2} = 2$ . Determine the capacitance for both configurations. [All dimensions are in meters]. [5 marks]

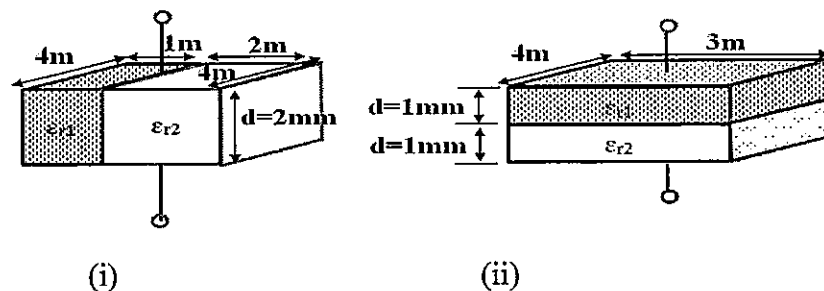


Figure 2(d)

Continued...

**Question 3 (25 marks)**

(a) State Ampere's Law and give its mathematical expression.

[4 marks]

(b) Figure 3(b) shows the cross section of a long coaxial cable. Current,  $I$ , flows along the positive  $z$ -direction in the inner conductor and returns through the outer conductor. The radius of the inner conductor is  $a$ , and the inner and outer radii of the outer conductor are  $b$  and  $c$ , respectively.

(i) Determine the magnetic flux density,  $B$  in each of the following THREE regions:  $r \leq a$ ,  $a \leq r \leq b$ , and  $b \leq r \leq c$ .

[12 marks]

(ii) Plot the magnitude of magnetic flux density,  $B$  as a function of  $r$ .

[3 marks]

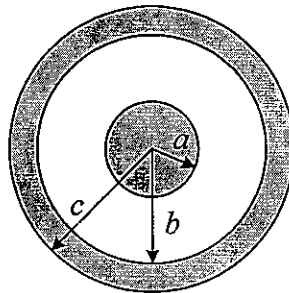


Figure 3(b)

(c) State ONE difference between electric force and magnetic force.

[2 marks]

(d) A charged particle with velocity  $\vec{u}$  is moving in a medium containing uniform fields  $\vec{E} = 20\hat{x}$  V/m and  $\vec{B} = 3\hat{y}$  Wb/m<sup>2</sup>. Determine  $\vec{u}$  so that the particle experiences no net force

[4 marks]

Continued ...

**Question 4 (25 marks)**

(a) Define the following terms

(i) Permeability

[2 mark]

(ii) Reluctance

[2 mark]

(iii) Coercive force

[2 mark]

(b) A magnetic circuit is made of three different materials as shown in Figure 4(b). The relative permeability of materials 1, 2 and 3 are 2000, 1000, and 800 respectively. Material 1 has a cross section area,  $A_1$  of  $(2 \times 2) \text{ cm}^2$  and a length,  $L_1$  of 10 cm. Material 2 has a cross section area,  $A_2$  of  $(2 \times 2) \text{ cm}^2$  and a length,  $L_2$  of 4 cm. Material 3 has a cross section area,  $A_3$  of  $(2 \times 2) \text{ cm}^2$  and a length,  $L_3$  of 6 cm. The coil carries a current,  $I = 0.5 \text{ A}$ , has 500 turns and an air gap of 1 mm is created at material 1. By using magnetic circuit concepts (assume that fringing effect is negligible),

(i) Sketch the equivalent circuit for the magnetic circuit of Figure 4(b).

[5 marks]

(ii) Determine the total reluctance of the circuit.

[10 mark]

(iii) Calculate the magnetic flux of the circuit.

[2 mark]

(iv) If a flux density of 0.4 Tesla is required at the air gap, what should be the new supplied current?

[2 marks]

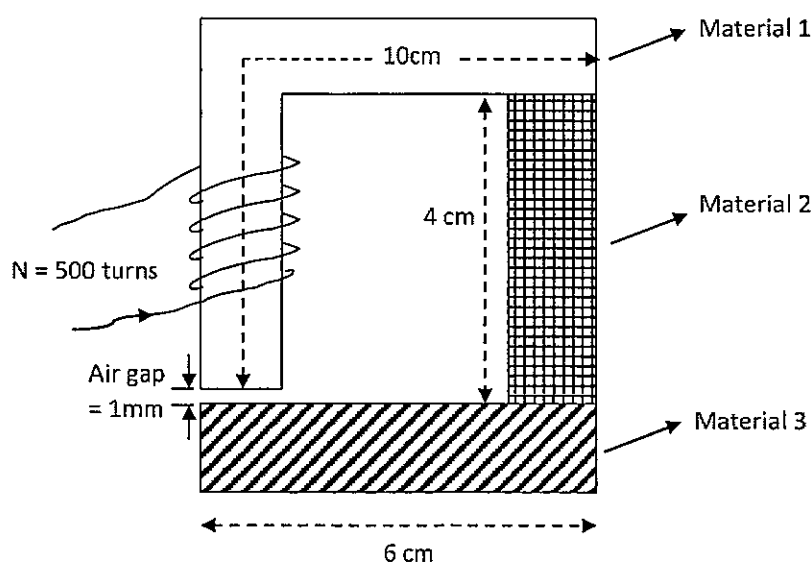


Figure 4(b)

**End of Paper**

**Appendix A: Physical Constants, Vector and Coordinate Transformations**

Elementary charge	$e$	$1.60 \times 10^{-19} \text{ C}$
Permittivity of free space	$\epsilon_0$	$8.85 \times 10^{-12} \text{ F/m}$
Permeability of free space	$\mu_0$	$1.26 \times 10^{-6} \text{ H/m} = 4\pi \times 10^{-7} \text{ H/m}$

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Differential length	$\hat{x}dx + \hat{y}dy + \hat{z}dz$	$\hat{r}dr + \hat{\phi}r d\phi + \hat{z}dz$	$\hat{R}dR + \hat{\theta}Rd\theta + \hat{\phi}R \sin \theta d\phi$
Differential surface areas	$d\vec{s}_x = \hat{x} dydz$ $d\vec{s}_y = \hat{y} dxdz$ $d\vec{s}_z = \hat{z} dxdy$	$d\vec{s}_r = \hat{r} r d\phi dz$ $d\vec{s}_\phi = \hat{\phi} dr dz$ $d\vec{s}_z = \hat{z} r dr d\phi$	$d\vec{s}_R = \hat{R} R^2 \sin \theta d\theta d\phi$ $d\vec{s}_\theta = \hat{\theta} R \sin \theta dR d\phi$ $d\vec{s}_\phi = \hat{\phi} R dR d\theta$
Differential volume	$(dx)(dy)(dz)$	$(dr)(r d\phi)(dz)$	$(dR)(R d\theta)(R \sin \theta d\phi)$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian & Cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x} \cos \phi + \hat{y} \sin \phi$ $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$ $\hat{z} = \hat{z}$	$A_r = A_x \cos \phi + A_y \sin \phi$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$ $A_z = A_z$
	$x = r \cos \phi$ $y = r \sin \phi$ $z = z$	$\hat{x} = \hat{r} \cos \phi - \hat{\phi} \sin \phi$ $\hat{y} = \hat{r} \sin \phi + \hat{\phi} \cos \phi$ $\hat{z} = \hat{z}$	$A_x = A_r \cos \phi - A_\phi \sin \phi$ $A_y = A_r \sin \phi + A_\phi \cos \phi$ $A_z = A_z$
Spherical & Cartesian	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z)$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta$ $\hat{\theta} = \hat{x} \cos \theta \cos \phi + \hat{y} \cos \theta \sin \phi - \hat{z} \sin \theta$ $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$	$A_R = A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi + A_z \cos \theta$ $A_\theta = A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi - A_z \sin \theta$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$
	$x = R \sin \theta \cos \phi$ $y = R \sin \theta \sin \phi$ $z = R \cos \theta$	$\hat{x} = \hat{R} \sin \theta \cos \phi + \hat{\theta} \cos \theta \cos \phi - \hat{\phi} \sin \phi$ $\hat{y} = \hat{R} \sin \theta \sin \phi + \hat{\theta} \cos \theta \sin \phi + \hat{\phi} \cos \phi$ $\hat{z} = \hat{R} \cos \theta - \hat{\theta} \sin \theta$	$A_x = A_R \sin \theta \cos \phi + A_\theta \cos \theta \cos \phi - A_\phi \sin \phi$ $A_y = A_R \sin \theta \sin \phi + A_\theta \cos \theta \sin \phi + A_\phi \cos \phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$
Cylindrical & Spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r} \sin \theta + \hat{z} \cos \theta$ $\hat{\theta} = \hat{r} \cos \theta - \hat{z} \sin \theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r \sin \theta + A_z \cos \theta$ $A_\theta = A_r \cos \theta - A_z \sin \theta$ $A_\phi = A_\phi$
	$r = R \sin \theta$ $\phi = \phi$ $z = R \cos \theta$	$\hat{r} = \hat{R} \sin \theta + \hat{\theta} \cos \theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R} \cos \theta - \hat{\theta} \sin \theta$	$A_r = A_R \sin \theta + A_\theta \cos \theta$ $A_\phi = A_\phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$

## Appendix B: Gradient, Divergence, Curl and Laplacian Operators

Cartesian coordinate (x, y, z)

$$\nabla V = \hat{x} \frac{\partial V}{\partial x} + \hat{y} \frac{\partial V}{\partial y} + \hat{z} \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x} \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{y} \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{z} \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinate (r,  $\phi$ , z)

$$\nabla V = \hat{r} \frac{\partial V}{\partial r} + \hat{\phi} \frac{1}{r} \frac{\partial V}{\partial \phi} + \hat{z} \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & r A_\phi & A_z \end{vmatrix} = \hat{r} \left( \frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{\phi} \left( \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \hat{z} \left[ \frac{\partial}{\partial r} (r A_\phi) - \frac{\partial A_r}{\partial \phi} \right]$$

$$\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate (R,  $\theta$ ,  $\phi$ )

$$\nabla V = \hat{R} \frac{\partial V}{\partial R} + \hat{\theta} \frac{1}{R} \frac{\partial V}{\partial \theta} + \hat{\phi} \frac{1}{R \sin \theta} \frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 A_R) + \frac{1}{R \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{R \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\begin{aligned} \nabla \times \vec{A} &= \frac{1}{R^2 \sin \theta} \begin{vmatrix} \hat{R} & \hat{\theta} & \hat{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & R A_\theta & (R \sin \theta) A_\phi \end{vmatrix} \\ &= \hat{R} \frac{1}{R \sin \theta} \left[ \frac{\partial}{\partial \theta} (A_\phi \sin \theta) - \frac{\partial A_\theta}{\partial \phi} \right] + \hat{\theta} \frac{1}{R} \left[ \frac{1}{\sin \theta} \frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R} (R A_\phi) \right] + \hat{\phi} \frac{1}{R} \left[ \frac{\partial}{\partial R} (R A_\theta) - \frac{\partial A_R}{\partial \theta} \right] \end{aligned}$$

$$\nabla^2 V = \frac{1}{R^2} \frac{\partial}{\partial R} \left( R^2 \frac{\partial V}{\partial R} \right) + \frac{1}{R^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{R^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}$$

**Appendix C: Table of Integrals**

$$\int \sin^2 \theta d\theta = \frac{\theta}{2} - \frac{1}{4} \sin 2\theta + \text{constant}$$

$$\int \cos^2 \theta d\theta = \frac{\theta}{2} + \frac{1}{4} \sin 2\theta + \text{constant}$$

$$\int \sin \theta \cos^2 \theta d\theta = -\frac{1}{3} \cos^3 \theta + \text{constant}$$

$$\int \cos \theta \sin^4 \theta d\theta = \frac{1}{5} \sin^5 \theta + \text{constant}$$

$$\int \sin 2\theta d\theta = -\frac{1}{2} \cos 2\theta + \text{constant}$$

$$\int x \sqrt{a^2 - x^2} dx = -\frac{1}{3} (a^2 - x^2)^{3/2} + \text{constant}$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \tan^{-1} \left( \frac{x}{\sqrt{a^2 - x^2}} \right) + \text{constant}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} + \text{constant}$$

$$\int \frac{dz}{(r^2 + z^2)^{3/2}} = \frac{z}{r^2 \sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{r dr}{(r^2 + z^2)^{3/2}} = \frac{-1}{\sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + \text{constant}$$

**End of Appendix**



